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Centre-of-mass and Pauli pair correlation correction to proton-nucleus total reaction cross section

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proton total reaction cross sections for ^{12}C, ^{16}O and ^{40}Ca in the energy range 0.02–1 GeV by considering the centre-of-mass and Pauli pair correlations in the target nuclei. It is found that the correlation correction, though not large, provides some improvement in the theoretical situation

Keywords Nuclear reaction, proton-nucleus total reaction cross section, pair correlation correction

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1. Introduction

Glauber multiple scattering model [1] is a fairly successful theoretical tool for analyzing nucleon-nucleus and nucleus-nucleus total reaction cross section data at medium and high energies (*e.g.* Refs. [2–4]). It is found to work fairly well even at relatively low energies, where its applicability is rather doubtful, provided that the model is suitably modified to account for the Coulomb force [4–7]. Confining the discussion to the p-nucleus system which concerns us here, a notable feature of almost all the applications of the model to the study of the total reaction cross sections is that they employ the so called optical limit approximation to the full Glauber S-matrix which consists of neglecting all but the leading term in the correlation expansion of the elastic S-matrix [8–10]. The leading optical limit term depends upon the one-body density of the target nucleus, while the neglected ones depend successively upon the two-body, three-body, correlations of the target nucleus. Calculations of p-nucleus elastic scattering differential cross sections around 1 GeV show that the contribution of the two-body pair correlation term is important enough to be included in any detailed analysis of the scattering data [9,10]. Although, the conclusions drawn from the study of the elastic scattering differential cross sections may not necessarily hold good to the case of p-nucleus total reaction cross sections because of the different sensitivities of the two types of the data to the details of the elastic S-matrix, still it is of some interest to see how the pair correlation term affects the calculated total reaction cross sections specially for two main reasons. First, apart from other factors, the two-body correlation term depends upon the square of the forward NN scattering amplitude $f_{NN}(0)$, hence it may have noticeable effect at low energies where $|f_{NN}(0)|$ assumes relatively large values. Second, the contribution of an important component of the two-body correlation function, namely the centre-of-mass (c.m.) pair correlation, though generally small, peaks in the nuclear surface region [10] which plays relatively more important role in the reaction cross section calculation than the interior region of the nucleus.

In this work, we present a calculation of p-nucleus total reaction cross section σ_R, in which the pair correlation term in the expansion of Glauber S-matrix is also considered. Since our main aim here is to study the importance of the pair correlation term in the calculation of σ_R rather than a detailed analysis of the available data, we consider a few cases only. In particular, we calculate σ_R for p-^{12}C, p-^{16}O and p-^{40}Ca collisions in the energy range 0.02–1.0 GeV. As will be seen below, the pair correlation contribution to the calculated σ_R depends greatly upon the incident proton energy and in general it improves the theoretical situation.

2. Theoretical consideration

In the Glauber model, the expression for σ_R for the collision of a nucleon on a target nucleus of mass number A, described by the ground state wave function ψ_0, may be written as

$$\sigma_R = 2\pi \int_0^\infty db\, b \left[1 - |S_{el}|^2\right] \tag{1}$$

with
$$S_{el}(b) = \left(\psi_0 \left| \prod_{i=1}^{A} \left[1 - \Gamma(b - s_i)\right] \right| \psi_0 \right), \tag{2}$$

where $\boldsymbol{b}$ is the impact parameter, $\boldsymbol{s}_i$ is the projection of the i-th target nucleon coordinate on the impact parameter plane and the NN profile function $\Gamma(b)$ is the two dimensional Fourier transform of the NN amplitude $f_{NN}(q)$. We will use the generally employed Gaussian parameterization [10] for $f_{NN}(q)$ in which case

$$\Gamma(b) = \frac{\sigma(1 - i\alpha)}{4\pi\beta}\, e^{-b^2/2\beta}. \tag{3}$$

In the above expression σ is the NN total cross section, α is the ratio of the real to the imaginary parts of the NN forward scattering amplitude and 2β is the range parameter.

There are several correlation expansion schemes for calculating $S_{el}(b)$ in terms of the target ground state density $\rho(r)$ and the pair correlation function $C(\boldsymbol{r}_1, \boldsymbol{r}_2)$ [8–10]. All give essentially same results. Here, we follow the effective profile expansion approach as developed in Ref. [10]. According to it, the expression for $S_{el}(b)$ after neglecting the 3-body and higher order correlation terms, reads as

$$S_{el}(b) \quad S_0(b) + \frac{A(A-1)}{2}(1 - \Gamma_0)^{A-2}\, I(b) \tag{4}$$

with
$$S_0(b) = \left[1 - \Gamma_0(b)\right]^A, \tag{5}$$

where
$$\Gamma_0(b) = \int \rho(r)\, \Gamma(b - s)\, dr \tag{6}$$

and
$$I(b) = \int C(r_1, r_2)\, \Gamma(b - s_1)\, \Gamma(b - s_2)\, dr_1\, dr_2. \tag{7}$$

The first term in eq. (4) gives the so-called optical limit approximation which combines all such multiple scattering terms in which the target nucleus always remains in the ground state. For sufficiently large A, it is a quite good approximation to write eq. (5) as

$$S_0(b) = e^{-A\,\Gamma_0(b)}, \tag{8}$$

which is the optical limit expression one obtains in the phase expansion approach [1]. Again for sufficiently large A, eq. (4) may be written as

$$S_{el}(b) = S_0(b)\left[1 + \frac{A(A-1)}{2}\, I(b)\right]. \tag{9}$$

Evaluation of $I(b)$ requires a knowledge of $C(r_1, r_2)$. In the literature, it is generally assumed that $C(r_1, r_2)$ is mainly contributed by three types of correlations. These are the c.m. and Pauli correlations which arise respectively due to the translational invariance and the antisymmetry of the target wave function, and the dynamical short range correlation which has its origin in the hard core of the NN interaction. The contribution of the last one is found to be negligibly small [9], hence it will not be considered here.

The problem of the c.m. and Pauli pair correlations for p-nucleus scattering has been discussed at length in Ref. [10] where an expression for $I(b)$ has also been derived following the approach of Feshbach *et al* [11] in the harmonic oscillator shell model for the target nucleus. The expression is

$$I(b) = I_{cm}(b) + I_{m}(b), \tag{10}$$

$$I_{cm}(b) = -\frac{1}{2\alpha_0^2 A}\left[\nabla \Gamma_0(b)\right]^2 \tag{11}$$

and
$$I_m(b) = \frac{1}{(A-1)}\left\{\Gamma_0^2(b) + I_{cm}(b)\right\} - \frac{3A\pi\sigma^2(1-i\alpha)^2}{8(A-1)} \int_{-\infty}^{\infty} dz\, \frac{\left[\rho^{(m)}(r)\right]^2}{k_F(r)\left[5 + 4\beta k_F^2(r)\right]}, \tag{12}$$

where α_0^2 is the oscillator constant, $\rho^{(m)}(r)$ is the model one-body density and $k_F(r)$ is the local Fermi momentum.

3. Calculation, results and discussion

The effects of the c m. and Pauli pair correlations on σ_R for p-^{12}C, p-^{16}O and p-^{40}Ca systems in the energy range 0.02–1.0 GeV have been studied using eqs. (1) and (8)–(11) The Coulomb force which plays and important role at lower energies is accounted for by employing the prescription of Ref. [7] which consists of multiplying the calculated σ_R by the factor $\left| 1 - \frac{44Z}{E_{cm} R_{int}} \right|$, where Z is the charge number of the target nucleus, E_{cm} is the c m energy in MeV and R_{int} is the interaction radius (defined by $\left|S_{el}\left(R_{int}\right)\right|^2 = 0.5$) in fm The quantity R_{int} is obtained using the Gaussian model for the ground state density of the target nucleus with the size parameter value as obtained from the charge rms radius after correcting for proton finite size (small variation in the value of R_{int} would not affect the main conclusions of this work). The quantity $S_0(b)$ which contribures dominantly to $S_{el}(b)$ of eq (9) is calculated using more realistic densities for the target nuclei. For this we first express $\Gamma_0(b)$ which occurs is eq (8) and is defined by eq. (6) in q-space as

$$\Gamma_0(b) = (1/2\pi ik)\int d^2q \, \exp\{-iq.b\} \, F(q) f(q), \tag{13}$$

where k is the incident nucleon momentum, $F(q)$ is the form factor of the ground state density of the target nucleus and $f(q)$ which is the Fourier transform of $\Gamma(b)$ is the NN scattering amplitude The advantage of working in q-space is that one can then directly use the sum-of-Gaussian parameterization of the nuclear form factor $F(q) = \sum_i a_i \exp[-b_i q^2]$ to evaluate $\Gamma_0(b)$ analytically. The values of a_i and b_i for ^{12}C and ^{40}Ca have been taken from Ref. [12] These values were determined by fitting the form factor as given by the charge densities of ^{12}C and ^{40}Ca of Refs [13] and [14] respectively, after correcting for proton finite size and assuming neutron and proton densities to be the same. The parameter values for ^{16}O which have been determined similarly, using the charge density of Ref [13] are : $a_1 = 1.72$, $a_2 = -0.72$, $b_1 = 0.874$ fm^2 and $b_2 = 0.539$ fm^2 These values reproduce the ^{16}O form factor upto $q = 2.5$ fm^{-1} quite well and of course the charge rms radius

To simplify the calculation of $I(b)$ as given by eqs. (10)–(12), we use the Gaussian model for the target ground state density This does not appear to be a poor approximation for the target nuclei considered in this work, specially for assessing the importance of the pair correlation correction in the calculation of the total reaction cross section which is the main aim of this work. Further, since the reaction cross section calculation are sensitive to the nuclear surface region where $k_l(r)$ assumes small values, the second term in the square brackets in the denominator of the integrand in eq. (12) may be neglected (this term is zero in the zero-range approximation used in Ref. [9]

With regard to the NN parameter values, we determine the values of σ from the parameterization of σ_{pp} and σ_{np} as given in Ref. [7] from where we also take the value of 2β. The parameter α is determined from the graphs for α_{pp} and α_{np} as given in Ref. [15].

The results of our calculation for the systems considered in this work are shown in Figure 1. The dashed curves are obtained in the optical limit approximation, while the solid ones show calculated σ_R values when both the c.m. and Pauli pair correlations are considered (The calculated σ_R values when only the c.m. pair correlation term is considered lie in between these two curves). From the Figure 1, it is seen that though the correlation correction is not large as it lies in the range of 0–4% for the cases considered in this work,

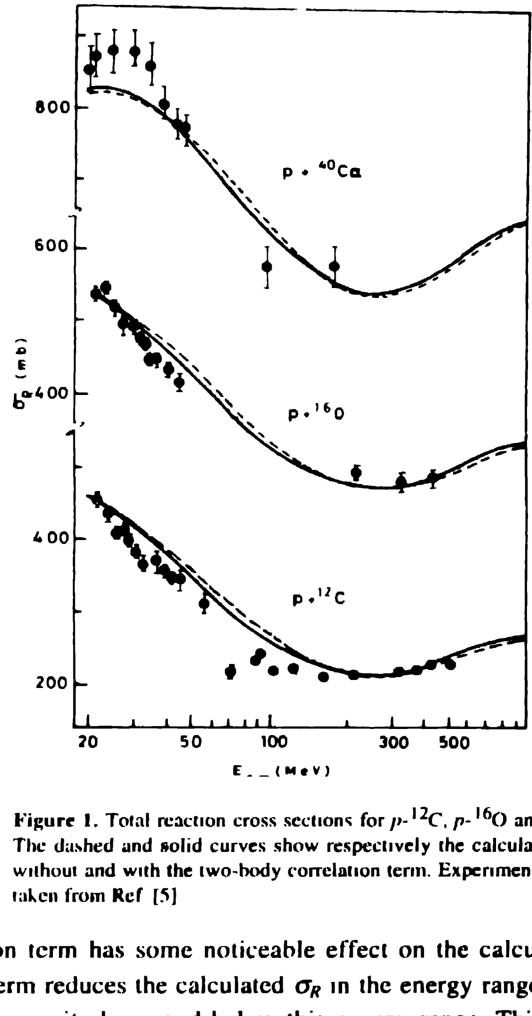

Figure 1. Total reaction cross sections for p-^{12}C, p-^{16}O and p-^{40}Ca systems. The dashed and solid curves show respectively the calculated cross sections without and with the two-body correlation term. Experimental data have been taken from Ref [5]

the correction term has some noticeable effect on the calculated σ_R. In each case, the correlation term reduces the calculated σ_R in the energy range of about 30–150 MeV and slightly enhances it above and below this energy range. This energy dependence of the correlation correction may be understood rather easily. To the first order in $I(b)$, the correlation correction is proportional to $\sigma^2(1 - \alpha^2)$. Thus, the correlation correction term assumes negligible values whenever $\alpha \simeq 1$ irrespective of the value of σ. This happens around 30 and 150 MeV which explains why the optical limit calculation and the calculation with the correlation term give same results around these energies. Similarly, the change in the sign of the correlation correction is a reflection of the sign change of the factor $(1 - \alpha^2)$ which assumes negative values in the energy range of about 30–150 MeV and positive values below and above this energy range. Further from Figure 1, it is also seen that the correlation correction works in the right direction as it brings the theoretical curve closer to the experimental values in the energy range $30 \leq E_{cm} \leq 150$ MeV without significantly affecting the agreement elsewhere. Thus, the consideration of the c.m. and Pauli correlations in the calculation of σ_R provides some improvement in the theoretical situation.

To conclude, the present study shows that the effect of the centre of mass and Pauli pair correlations on the calculated σ_R does not exceed 4% level in the energy range considered in this work. However, the correlation correction works in the right direction as it provides some improvement in the theoretical situation. Refinements of the correlation calculation such as invoking the energy dependence of the parameter β, including the dynamical short range correlation, *etc*. are unlikely to affect the present conclusions in any substantial way.

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